

CATALOG INFORMATION

Dept and Nbr: MATH 6                      Title: INTRO TO HIGHER MATH  
Full Title: An Introduction to Higher Mathematics  
Last Reviewed: 12/12/2023

Units		Course Hours per Week		Nbr of Weeks	Course Hours Total	
Maximum	4.00	Lecture Scheduled	4.00	17.5	Lecture Scheduled	70.00
Minimum	4.00	Lab Scheduled	0	6	Lab Scheduled	0
		Contact DHR	0		Contact DHR	0
		Contact Total	4.00		Contact Total	70.00
		Non-contact DHR	0		Non-contact DHR	0

Total Out of Class Hours: 140.00

Total Student Learning Hours: 210.00

Title 5 Category: AA Degree Applicable  
Grading:                      Grade Only  
Repeatability:            00 - Two Repeats if Grade was D, F, NC, or NP  
Also Listed As:  
Formerly:

**Catalog Description:**  
Students will be introduced to topics in higher mathematics including introductory set theory and formal logic, proof techniques, mathematical induction, equivalence relations, functions and cardinalities of sets as applied to number theory, calculus, and modern algebra.

**Prerequisites/Corequisites:**  
Course Completion of MATH 1B

**Recommended Preparation:**

**Limits on Enrollment:**

**Schedule of Classes Information:**  
Description: Students will be introduced to topics in higher mathematics including introductory set theory and formal logic, proof techniques, mathematical induction, equivalence relations, functions and cardinalities of sets as applied to number theory, calculus, and modern algebra.  
(Grade Only)  
Prerequisites/Corequisites: Course Completion of MATH 1B  
Recommended:

Limits on Enrollment:

Transfer Credit: CSU;UC.

Repeatability: Two Repeats if Grade was D, F, NC, or NP

## **ARTICULATION, MAJOR, and CERTIFICATION INFORMATION:**

<b>AS Degree:</b>	<b>Area</b>			Effective:	Inactive:
<b>CSU GE:</b>	<b>Transfer Area</b>			Effective:	Inactive:
<b>IGETC:</b>	<b>Transfer Area</b>			Effective:	Inactive:
<b>CSU Transfer:</b>	Transferable	Effective:	Spring 2013	Inactive:	
<b>UC Transfer:</b>	Transferable	Effective:	Spring 2013	Inactive:	

### **CID:**

### **Certificate/Major Applicable:**

Major Applicable Course

## **COURSE CONTENT**

### **Student Learning Outcomes:**

At the conclusion of this course, the student should be able to:

1. Identify valid forms of arguments using predicate logic.
2. Construct mathematical proofs of theorems in number theory, calculus, and modern algebra.
3. Determine the properties of functions and relations.
4. Determine the cardinalities of important sets.

### **Objectives:**

At the conclusion of this course, the student should be able to:

1. Use set theory and logic to convey and understand mathematical concepts.
2. Prove and disprove mathematical conjectures.
3. Provide proper counterexamples.
4. Prove mathematical theorems using mathematical induction and strong mathematical induction.
5. Characterize functions as injective, surjective, bijective, or otherwise.
6. Characterize relations in terms of the reflexive, symmetric and transitive properties.
7. Determine when a relation is an equivalence relation.
8. Characterize a set in terms of its cardinality.
9. Apply techniques of proofs to number theory, calculus, and topics in modern algebra.

### **Topics and Scope:**

#### **I. Logic**

- A. Negation, conjunction, and disjunction
- B. Logical form and equivalence
- C. Conditional statements
- D. Biconditional statements
- E. Valid and invalid arguments
- F. Predicates

- G. Quantified statements
- H. Arguments with quantified statements
- II. Set Theory
  - A. Introduction to sets
  - B. Algebra of sets
  - C. Venn diagrams and conjectures
  - D. Arbitrary unions and intersections
- III. Proofs
  - A. Trivial and vacuous proofs
  - B. Direct proofs
  - C. Indirect proofs
    - 1. Proof by contradiction
    - 2. Proof by contrapositive
  - D. Existence proofs
  - E. Counterexamples
  - F. Proofs by cases
- IV. Mathematical Induction
  - A. Introduction to mathematical induction
  - B. Proof by minimum counterexample
  - C. Strong mathematical induction
- V. Functions
  - A. Definition
  - B. Injective and surjective functions
  - C. Bijective functions
  - D. Composition of functions
  - E. Inverse functions
  - F. Permutations
- VI. Relations
  - A. Relations on sets
  - B. Properties of relations
  - C. Equivalence relations
  - D. Equivalence classes
  - E. Congruence modulo  $n$
  - F. Integers modulo  $n$
- VII. Cardinalities of Sets
  - A. Numerically equivalent sets
  - B. Denumerable sets
  - C. Uncountable sets
  - D. Comparison of set cardinalities
- VIII. Topics in Number Theory
  - A. Divisibility properties of integers
  - B. The division algorithm
  - C. Greatest common divisors
  - D. The Euclidean algorithm
  - E. Relatively prime integers
  - F. The Fundamental Theorem of Arithmetic
  - G. Concepts involving sums of divisors
- IX. Topics in Calculus
  - A. Limits of sequences
  - B. Infinite series
  - C. Limits of functions and fundamental properties
  - D. Continuity

- E. Differentiability
- X. Various Topics from Modern Algebra that Could Include:
  - A. Binary operations
  - B. Groups
  - C. Rings
  - D. Fields
  - E. Integral domains
  - F. Fundamental properties
  - G. Isomorphism
  - H. Homomorphisms
  - I. Subgroups

### Assignment:

1. Daily reading outside of class (10-50 pages per week).
2. Homework assignments (15-30; with 5-35 math problems) from:
  - A. Required text(s)
  - B. Supplementary materials chosen by the instructor.
3. Quiz(zes) (0-8) and exams (2-6), including final exam.
4. Project(s) (0-3): research papers on a specific topic (5-10 pages) or presentations given as posters or short talks. Papers and presentations must be related to topics taught in the course.

### Methods of Evaluation/Basis of Grade:

**Writing:** Assessment tools that demonstrate writing skills and/or require students to select, organize and explain ideas in writing.

None, This is a degree applicable course but assessment tools based on writing are not included because problem solving assessments are more appropriate for this course.

Writing  
0 - 0%

**Problem Solving:** Assessment tools, other than exams, that demonstrate competence in computational or non-computational problem solving skills.

Homework assignments

Problem solving  
5 - 20%

**Skill Demonstrations:** All skill-based and physical demonstrations used for assessment purposes including skill performance exams.

None

Skill Demonstrations  
0 - 0%

**Exams:** All forms of formal testing, other than skill performance exams.

Quiz(zes) and exams

Exams  
70 - 95%

**Other:** Includes any assessment tools that do not logically fit into the above categories.

Projects: research papers or presentations

Other Category  
0 - 10%

**Representative Textbooks and Materials:**

Mathematical Proof: A Transition to Advanced Mathematics. 4th ed. Chartrand, Gary and Polimeni, Albert and Zhang, Ping. Pearson. 2023.

An Introduction to Higher Mathematics. Keef, Patrick and Guichard, David. Creative Commons. 2021.

A Concise Introduction to Pure Mathematics. 4th ed. Liebeck, Martin. Routledge. 2017 (classic)

Introduction to Advanced Mathematics. 2nd ed. Barnier, William and Feldman, Norman. Pearson. 2000 (classic)